An Objective Approach to Relative Valuation[1,2]

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Abstract

A fundamental approach to describing the behaviour of the equity price index is presented. The method centres on the contention that, under a constant discount rate and in a market that is efficient and in equilibrium, the forward-looking risk premium is equivalent to the expected dividend yield, and both are equal to zero. Extending this special-case scenario to one that involves time-wise variations in the discount rate leads to a special co-ordinate transformation [*or mapping*], *which addresses how the index should behave correspondingly.*

Applying the same principle to both, corporate earnings and the nominal gross domestic product [*GDP*], *leads to a similar transformation. This, consequently, makes way for objective comparisons between the equity index, corporate earnings and the GDP, thereby raising the notion of relative valuation in this context. A practical demonstration of this is ultimately provided for the US and UK economies and equity markets.*

1. Introduction

Relative valuation is an important concept in investment and finance. Its significance is observed in day-to-day investment activities, where gaps or spreads in yields, interest rates and other rates of growth, in general, are exploited (Fabozzi, 1999).

The main advantage of relative valuation over ad-hoc approaches is that it paves the way for unbiased comparisons – i.e. given certain stocks, which one(s) should one invest in, or is the equity index over/undervalued relative to what the earnings, GDP, etc. indicate. In this context, therefore, relative valuation eliminates the need for an absolute measure, which is, arguably, an impossible feat to achieve.

Relative-valuation measures typically involve multiples, and these have been applied time after time to comparing, among others, growth stocks (Peters, 1991), bonds (Benari, 1988), funds (De Long and Shliefer, 1992), as well as indices across different countries (Arnott and Henricksson, 1989). In addition, such measures have even been used to provide a process by which financial health could be assessed (Barth *et al*, 1998). Given that the above represent only a small fraction of the literature on how and why relative valuation is put into use, it is, therefore, inevitable that this notion has a lot to offer when it comes to practical investment.

Here, as well, we intend to apply the notion of relative valuation for comparison purposes. Our aim, as it turns out, is to propose and develop an objective way for comparing three basic elements with each other - namely, the state of the economy, as represented by the nominal GDP, corporate earnings and, finally, the equity price index. Multiples representing these are most likely available and used commonly by both, economists and investment/financial analysts, to tie together the economy and the stock market. Such a measure of relative valuation could, thus, enable one to assess whether the

[1] Working paper.
[2] April/May 2000.
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stock market is over, under or fairly valued in comparison with earnings and/or GDP.

Our primary concern here is to derive a fundamental method for describing the time-dependent behaviour of the stock market, earnings and the GDP in relation to changes in the rates of discount. What these discount rates encompass – i.e. short or long-term bond or other types of yields and interest rates – and how implementing different ones should alter the final results, are important issues that will be discussed as we go along.

In the process, we shall note two attributes that might raise doubts on our methods and conclusions. Firstly, the relationships derived here will look different from those typically found in the literature. This is because ours are extracted from an approach that follows a different route altogether. Secondly, the issue of spurious relations could be raised. To overcome this, we will try our best to supply proofs and charts at every step of the way. In addition, we shall keep the paper as objective as possible by limiting it to factual observations and leaving out any hypothetical and speculative explanations.

2. The Approach

We plan to develop the method in two ways – one focusing on equity [Section 2a] and the other on GDP and corporate earnings [Section 2b]. The latter two occupy the same section because their underlying principles happen to be the same. The final results will then be combined together to establish the relative valuation measures. For sake of brevity, all derivations that already exist in the literature cited shall be omitted.

2a. The Equity Model

It is useful to begin with our previous contention that, in a constant-discount rate environment and where the market is efficient and at steady-state equilibrium, the equity risk premium is zero (Cohen, 2000). This clearly represents a highly idealised scenario, but nonetheless it can and will be generalised here to account for time-wise changes.

Let us begin by recalling that in the perfect state defined above, all rates of growth – that is, in price, dividends and earnings – will be equal to each other, as well as to a constant. This, obviously, leads to the zero-dividend-yield criterion.

It can further be shown that upon letting the discount rate, R, be equal to this presumed constant, R^*, we obtain the following expression:

$$\left(\frac{\partial \ln S}{\partial t}\right)_{R=R^*=constant} = R^* \qquad (1)$$

where S is the price and the quantity on the left-hand side represents the percent rate of growth in price [i.e. capital gains], conditional on the discount rate being constant at R^* - that is $R = R^* = constant$. Note that Equation 1 is simply the market's rate of return, incorporating the zero-dividend-yield condition discussed earlier. It thus follows from 1 that the [logarithm of] price, $\ln S$, could be written as a function of time, t, as well as R^* - that is:

$$\ln S = \ln S(R^*, t) \qquad (2)$$

Holding the discount rate constant in the above clearly imposes a daunting constraint on S. This, however, may easily be relaxed with the help of a simple mathematical procedure, which entails the notion of the exact differential. The details of this procedure shall be left out from here simply because they are available in almost any intermediate-level text on differential calculus.

Very briefly, the approach is as follows. In place of writing $\ln S(R^*,t)$ as we have done in 2, we express it as

$$\ln S = \ln S(R, t) \qquad (3)$$

which generalises S to account for a time-variable discount rate, $R = R(t)$, instead.

The rationale behind Equation 3 is that the effects of the market, and the economy in general, on S are presumed to enter separately through two fundamental elements, one which is R and the other, which comprises everything else that falls outside the reign of R. As the second variable appears as time, t, it renders Equation 3 general and, hence, along with $R(t)$, it should capture all the economic and market effects on the price, S. In other words, expressing S in the form of Equation 3 effectively removes all the restrictions imposed on it earlier.

In light of the above, we take the total time differential of Equation 3 and obtain:

$$\frac{\Delta \ln S(R,t)}{\Delta t} = \left(\frac{\partial \ln S}{\partial t}\right)_R + \left(\frac{\partial \ln S}{\partial R}\right)_t \frac{\Delta R}{\Delta t} \quad (4)$$

While the first partial differential – i.e. $(\partial \ln S / \partial t)_R$ - has been shown to be equal to R [see Equation 1], the second, $(\partial \ln S / \partial R)_t$, is simply the stock duration, which is the sensitivity of the price to changes in the discount or interest rate at some point in time. The second term in full, which also includes variations in the discount rate, embodies the risk premium as well.

Being a differential of an exact function, therefore, the two components in Equation 4 are coupled to each other via:

$$\left(\frac{\partial}{\partial R}\left(\frac{\partial \ln S}{\partial t}\right)_R\right)_t = \left(\frac{\partial}{\partial t}\left(\frac{\partial \ln S}{\partial R}\right)_t\right)_R = 1 \quad (5)$$

which incorporates also Equation 1. This could be integrated twice to yield the general solution to Equation 4, whereby

$$\ln S = Rt + \alpha_0 + \alpha_1 R + \tilde{\Psi}(R) \qquad (6)$$

with α_0 and α_1 being integration constants and $\tilde{\Psi}(R)$ a yet unknown function of only *R*.

Alternatively, we may recast 6 as:

$$\ln S - Rt = \alpha_0 + \Psi(R) \qquad (7)$$

where $\Psi(R)$ is another function of *R*. The latter representation conveniently absorbs both $\tilde{\Psi}(R)$ and $\alpha_1 R$ into a single function, $\Psi(R)$.

It thus follows from 7 that plotting $\ln S - Rt$ against *R* should, in theory, produce a single curve, depending only on *R*. This transformation, as a result, brings in all the effects of time on $\ln S - Rt$ through *R*. A schematic illustration is presented in Figure 1, where a mapping of *S versus R* into $\ln S - Rt$ *versus R* is shown to introduce some type of regularity to a relatively disordered graph.

As per our derivation so far, we find it necessary to mention two points. First, even though Equation 7 is extracted from what appears to be formidable and perhaps too theoretical an approach, it is indeed very easy to apply and, also, as it shall be demonstrated shortly, it does possess real and practical uses. Second, questions relating to the discount rate have undoubtedly been raised by now. For instance, what is the discount rate, how should it be defined and, more importantly, how should one deal with it? The answer to these, as it will turn out in Section 3, happens to be surprisingly straightforward. Beforehand, though, let us go ahead and apply the same logic to both, the nominal GDP and earnings.

2b. Applications to GDP and Earnings

It is well accepted that movements in the equity price index are closely tied to corporate earnings and, even more generally, to the economy. Common sense further dictates that a bull market typically comes with a strong economy and a bear market with a weak economy. One probable explanation for this is simply that the market comprises a subset of the economy – i.e. corporate earnings constitute a [small] fraction of the GDP. This, therefore, should enable one to derive a GDP relationship that is analogous to the one for equity, as well as for corporate earnings.

Before we begin, however, we need to introduce a couple of analogies to the equity price index. This is possible with the help of the earnings discount model[4]. For this, let us define V_G and V_E, respectively, as the "values" associated with the nominal GDP and corporate earnings. Based on the above, therefore, V_G could be represented by

$$V_G(t) \equiv \frac{G_f(t)}{R} \qquad (8a)$$

and V_E by

$$V_E(t) \equiv \frac{e_f(t)}{R} \qquad (8b)$$

[4] Conditions under which the earnings discount model becomes as valid as the dividend discount model have been discussed earlier (Cohen, 2000). This entails an efficient market in which the firm-generated valuation equals that of the investor's. Accordingly, therefore, the approach introduced in the present paper will not be suitable to pre 1950, where this type of behaviour was not followed, at least by the S&P index.

where $G_f(t)$ and $e_f(t)$, respectively, are the time-t expectation of the nominal GDP and corporate earnings one year ahead, at t+1. Thus, with R being the discount rate, an earnings-discount-type valuation model is being imposed on the economy as well.[5] It should further be stressed that the one-year-ahead nominal GDP, i.e. $G(t+1)$, will from now on be implemented instead of the expected purely for convenience, as we shall assume that the two are equal in an information-efficient economy. For corporate earnings, on the other hand, Datastream's aggregate I/B/E/S forecasts will be presumed sufficient for our purposes.

Now, with the above analogy in place, it is not difficult to demonstrate that the same rules that dominate the price index should apply to V_G and V_E as well, yielding expressions similar to Equation 7, but with V_G and V_E substituted for S. This, consequently, leads to:

$$\ln V_G - Rt = \beta_0 + \Phi(R) \qquad (9a)$$

and

$$\ln V_E - Rt = \gamma_0 + \Xi(R) \qquad (9b)$$

where β_0 and γ_0 are integration constants and $\Phi(R)$ and $\Xi(R)$ are functions of R only.

Aside from noting that the same transformation that presides over the equity model applies to here as well, $\Phi(R)$ and $\Xi(R)$ may not necessarily be the same as $\Psi(R)$. A comparison of these shall be undertaken later in Section 6, however we need to address certain issues beforehand, namely of the discount rate, "reversibility" and "structural or regime shifts."

3. The Discount Rate

As mentioned at the end of Section 2a, the issue of the discount rate is an important one. Putting it more precisely, what should one use for R in Equations 7 and 9 in order to be able to test their validity?

Obviously, several choices exist. These include some measure of risk premium added to some short/long-term risk-free interest rate, bond yield, etc. Clearly, therefore, this makes room for a lot of subjectivity. But, never the less, we will attempt to settle this for our purposes here.

Let us begin by recalling that, in a perfect market and conditional on $R = R^* = constant$, all discount rates and rates of return will be constant and equal to each other. Moreover, because of the risk premium being equal to zero, these will all converge to the rate of interest, which is also constant.

The zero-risk-premium constraint, which eliminates all uncertainties on future projections, will additionally render all yield and term structure curves flat by removing the spreads between the long and short-term interest rates and yields. We, therefore, will have remaining only one constant rate of interest, which we shall denote by r^*. All this, of course, sets up the stage for a highly idealised, as well as unrealistic, base-case scenario, which clearly extends the Golden rule of economics to finance.

Bearing this in mind, it will be useful to hypothesise that any real-life scenario could be considered as merely a perturbation away from the base case. This line of reasoning will prove to be important here, as it will enable us to modify the highly superficial situation to a real one.

To deal with this, let us refer to either Equation 7 or 9. For convenience, we shall work with Equation 7, although the logic that follows could equally as well apply to both, 9a and 9b.

Let us begin with the assumption that this idealised, base-case scenario, where the interest rate is equal to the constant discount rate, is in place. In addition, recall that the left-hand side of Equation 7, which is $\ln S - Rt$, is a function of the <u>single</u> parameter, R, i.e. $\Psi(R)$, the implication of which is schematically presented in Figure 1. Putting these together gives

$$\ln S - R^* t = \alpha_0 + \Psi(R^*) \qquad (10)$$

and

$$\ln S - r^* t = \alpha_0 + \Psi(r^*) \qquad (11)$$

Moreover, the condition $R^* = r^*$ makes way for the equality

[5] Note that if we let R in Equation 8b be the US government 10-year nominal bond yield, we effectively recover the Federal Reserve's valuation model (Greenspan, 1997).

$$\ln S - R^{*}t = \ln S - r^{*}t \qquad (12)$$

simply because

$$\Psi(R^{*}) = \Psi(r^{*}) \qquad (13)$$

We now move away from this idealised state by introducing time-variable rates instead. This can be accomplished by perturbing R^* by some increment ΔR and r^* by Δr, whereby both, ΔR and Δr are free to vary in time, *t*. This, consequently, modifies Equations 10 and 11 to:

$$\ln S - [R^{*} + \Delta R]t = \alpha_0 + \Psi(R^{*} + \Delta R) \qquad (14)$$

and

$$\ln S - [r^{*} + \Delta r]t = \alpha_0 + \Psi(r^{*} + \Delta r) \qquad (15)$$

respectively. It is important to recognise that, because the function Ψ is dependent only on the single parameter - be it *R* or *r* - the effect of time on Ψ enters indirectly through either, *R* or *r* - i.e. $R(t)$ and $r(t)$.

Generalising this scenario even further by choosing two different points in time - i.e. t_1 and t_2 - such that $\Delta R(t_1)$ equals $\Delta r(t_2)$, enables us to equate 14 and 15 and get:

$$\ln S - [R^{*} + \Delta R]t_1 = \ln S - [r^{*} + \Delta r]t_2 \qquad (16)$$

simply because $R^{*}+\Delta R$ is chosen to be equal to $r^{*}+\Delta r$. The significance of this is that it makes no difference, whatsoever, as to what one incorporates for *R* in Equations 7 and 9. For this matter, *R* could be either the discount rate, if known, or any of the several available choices for the interest rate, be it short term or long.

The above could be observed in mappings that incorporate yields of government bonds with different maturities. Data convergence can be detected in virtually all cases, although, for sake of brevity, we have decided not to include any examples here.[6] Having said this, we now go on to discuss the concepts of reversibility and structural shifts, and how they enter this work.

[6] The interested reader could obtain these directly from the author.

4. Reversibility and Structural Shifts

The equity price, GDP and earnings representations provided in Equations 7 and 9a-b lead to some important conclusions regarding "reversibility" and "structural shifts." Realising that structural shifts tend to alter the behaviour of the economy and the markets, an important objective here, as in any economic and financial analysis, would thereby consist of defining ways for detecting and, possibly, classifying them.

To carry this out here, we start with the observation that $\ln S - Rt$, $\ln V_G - Rt$ and $\ln V_E - Rt$ must depend solely on *R* via the functions $\Psi(R)$, $\Phi(R)$, and $\Xi(R)$, respectively. The effects of time, as mentioned earlier, enter indirectly through *R*. Whether or not this functional dependence of *R* on *t* is the same in all situations is not of concern now, but, in any case, it shall be dealt with shortly.

An important outcome of such dependence is the notion of "reversibility," which may be explained as follows. Take, for instance, Figure 2, where we focus on the time-dependent behaviour of one of UK benchmark government bond yields, namely the 10-year, as obtained from Datastream. We could have very well selected other yields instead, as the choice makes no difference to what follows hereafter. The time frame here covers the period Q1-80, to Q4-99.

Let us now, for the sake of example, identify and highlight 7 points where the yield crossed the value of 10%, all occurring on different dates between 1986 and 1992. Very clearly, even though the yields were identical on these dates, which fall some years apart, one does not expect the corresponding values for *S*, GDP and corporate earnings to remain in any way the same. Strictly speaking, therefore, *S*, GDP and corporate earnings, by themselves, are "irreversible" functions of *R*, as they obviously tend to move in ways different than *R*.

The concept of reversibility, never the less, comes into play when we consider, instead, the mapping prescribed by Equations 7 and 9. Here we expect the transformed relations, $\ln S - Rt$, $\ln V_G - Rt$ and $\ln V_E - Rt$ to reposition with *R*, as the equations suggest. Therefore, if *R* were, say, equal to 10% in 1986, varies randomly over time and in 1991, about five years later, reverts back to 10%, the transformed relations should also revert back to their original 1986 values. This concept, which, in a sense, forces us to move away from the notion of time series, is also schematically presented in Figure 1. The consequence of such behaviour shall be referred to as reversibility and will play a dominant role in our work.

Alternatively, a structural or regime shift implies the contrary. If, for instance, a plot of the above-mentioned points fall on notably disparate lines, then a structural shift, separating these points, might have occurred in between. Schematically, a structural shift is exemplified in Figure 3, where mapping *S versus R* into $\ln S - Rt$ *versus R* over a given time frame leads to distinctive characteristic patterns. Empirical evidence of both phenomena, namely reversibility and regime shift, will be provided next.

4a. Evidence of Reversibility and Structural Shifts in the UK

We have argued so far that one could incorporate any of the available interest rates in Equations 7 and 9a-b. If our hypothesis were correct, then on plotting $\ln X - Rt$ against *R*, where *X* could equally represent *S*, V_G or V_E, we should expect to obtain a single curve, or, more generally, a series of curves, each pertaining to some particular structural regime in the market and/or the economy. Although this mapping technique shall be applied here only to the US and the UK, we should emphasise that, in addition, a number of other economies and markets tend to display similar behaviours (Cohen and Chibumba, 1999).

First, let us go to Figures 4-6, which illustrate $\ln S - Rt$, $\ln V_G - Rt$ and $\ln V_E - Rt$, respectively, plotted against *R* for the FTSE 100 index and the UK economy. We have implemented here for *R* some nominal benchmark government bond yield. This is being utilised purely for consistency, because Datastream provides similar series for a number of other markets as well. It should also be mentioned that all data are quarterly, beginning in 1980, which is when Datastream started to provide them, to the present. Moreover, we note that frequency is not an issue since it does not alter any of the results that follow next.

We refer to Figures 4a and 4b, which depict *S* and $\ln S - Rt$, respectively, plotted against *R*. Again, for *R* we have incorporated the above-mentioned 10-year yield, even though similar conclusions apply as well to the other yields.

On comparing Figure 4a to 4b, each also containing a best-fit cubic-polynomial curve for reference, we do observe a convergence of data where the proposed transformation has been applied. From a statistical perspective, we implement the standard deviation relative to the best fit as a measure of the scatter. This yields 0.247 and 0.088, respectively, belonging to Figures 4a and 4b. Clearly, therefore, the mapping does introduce a significant amount of data convergence, consistent with the prediction.

Although in this case the mapping reduces the scatter [in terms of standard deviation by a factor of *ca.* 3], we observe that still not all the scatter has been eliminated. This, we believe, is owed to the fact that the index comprises various types of investors/firms, each providing a valuation based on a different discount rate. Therefore, aggregating all these together into a single equity price, *S*, should naturally create a dispersion in the mapped plane.

We have, in addition, highlighted in both figures the data for the 3 quarters immediately preceding the October 1987 stock market crash. It is remarkable that while there is virtually no indication of “over pricing” in Figure 4a, the phenomenon becomes clearly visible as ouliers once the mapping is imposed. This is clearly portrayed in Figure 4b. Interestingly also, a similar observation, although less conspicuous but equally prominent, applies to the S&P 500 market as well [see Figure 10]. In view of this, therefore, the methodology could potentially be useful as a means for detecting over/under pricing. However, being highly speculative at this point, there is need for more rigorous testing before this claim could be confirmed.

Returning now to the analysis, we note that the scatter in Figure 4b is even more diminished if we were to graph $\ln V_G - Rt$ *versus R* instead, with *R* again representing a bond yield. This is shown in Figure 5, where data convergence is much more noticeable than in the previous case. This, we presume, is because V_G takes into account the differences in investors – that is, in arriving at

V_G, an investor who, let us suppose, discounts at 5% will compute a V_G different from one that discounts at, let us say, 10%.

Last, but not least, the same logic applies to earnings as well, which is graphed against the nominal 10-year yield in Figures 6a and 6b, both alone and transformed, respectively. Note once again the marked data convergence in Figure 6b, where the mapping $\ln V_E - Rt$ is implemented instead.

As for reversibility and structural shifts, they are evident in all the graphs, more so in Figures 5 and 6b, where streak-like patterns emerge. Each of these streaks corresponds to what we believe to be a distinct structural regime. Samples of these are circled in Figures 5 and 6b, which, for convenience, are also shown expanded in Figures 7 and 8. We note here that data convergence is indeed remarkable under the proposed transformation. This, therefore, strongly supports the underlying hypothesis regarding reversibility and structural shifts.

It thus follows that reversibility occurs along any of the streaks in Figures 5 and 6b, where movements in interest rates, in some cases over many years, do not appear to throw any of the data points out of its course. This, presumably, happens because all these points belong to a distinct structural regime. A more detailed assessment of this will follow once we examine the situation for the US.

4b. Evidence of Reversibility and Structural Shifts in the US

Figures 10-12 depict the three parameters, namely, $\ln S - Rt$, $\ln V_G - Rt$ and $\ln V_E - Rt$, all plotted against R. We observe that, as in the UK's case, the relevant conclusions are identical. These are basically (1) all graphs exhibit data convergence, and (2) reversibility and regime shifts are again evident, particularly in Figures 11 and 12 where the scatter is markedly less.

Apart from these similarities with the UK, an important feature further manifests itself in Figures 10 and 12, where prices and earnings are concerned. In both, there appears to be a branching of data, especially at yield rates lower than 7%. One of the branches, which is highlighted and whose time frame is indicated, certainly belongs to a distinct structural regime. We shall return to this when we discuss both, the role of relative valuation in, and its relationship to, this work.

5. Relative Valuation

Within the framework of our analysis, relative-valuation measures could be arrived at by simply superimposing the data in Figures 4-12. The outcome of this shall be demonstrated here for both the UK and US. We should warn, however, that this is not an exercise in "forecasting." It is merely a methodology by which intrinsic values could be compared objectively against one another.

It is also useful to mention that while the conclusions that follow are based more on visual comparisons, an in-depth statistical analysis is necessary to put these findings on more solid grounds. This, however, will come in later in Section 6 after the initial concept is laid out.

5a. Relative Valuation in the UK

The relative-valuation measures for the UK are examined here by overlaying Figures 5-7 on top of one another. For example, superimposing Figure 5 on 4b depicts how the FTSE 100 price index matches against the economy, both historically and currently. Similarly, laying Figure 6b on 4b and 6b on 5, respectively, illustrates how the economy compares against the equity market and corporate earnings.

Let us begin with Figure 13, which superimposes the price index on the nominal GDP, both in their special co-ordinate transformations. We note here that, over the long run, the two parameters appear to move together. This provides some justification to the principle that, in the long term, equity prices and the GDP are related. Moreover, Figures 14 and 15, respectively, which overlay the price index on the earnings and the earnings on the GDP, tell a similar story. Here, as well, the three elements, at least in the UK, appear to be in balance, moving very much together in the long run.

5b. Relative Valuation in the US

Section 5a summarised the situation for the UK. The US, as we shall see now, leads to some different conclusions.

Figure 16 displays an overlay of the S&P 500 price index over the GDP. We note here that from 1980, which is the start of the data, to 1990, the points fall, more or less, on top of each other. This basically indicates that during this period, the two parameters were moving in tandem with one another, as economic theory dictates.

In contrast to the above, however, we observe that a breakdown in the relationship occurs somewhere in the middle, when the two begin to go in separate directions. The deviation begins sometime in 1995, when the S&P 500 index and the GDP appear to move along different paths altogether. Whether this means that the index is overpriced relative to the GDP and/or the components of the index have, on aggregate, been outperforming the average economy is perhaps speculation. But, whatever the reason, the insinuation is that from 1995 onwards, the S&P 500 market has failed to represent the economy, both adequately and fairly.

Figure 17 illustrates the S&P 500 superimposed over corporate earnings. The situation in this case is somewhat clearer than in the previous figure. Here, for instance, we observe that the post-1994 increases in equity prices have followed the sudden jump in the corporate earnings seen in Figure 12. Even here, there seems to be a post-1994 out-performance of the price index relative to earnings.

Finally, we superimpose corporate earnings over the GDP in Figure 18. Once again, avoiding all hypothetical explanations and relying strictly on visual comparison[7], we note that until around the beginning of 1995, corporate earnings moved together with nominal GDP. Past that period, however, a shift in corporate earnings caused it to outperform the GDP, with the latter still continuing in its pre-1995 course. This is consistent with our observation in Figure 16, where we concluded that, post 1994, the S&P 500 equity market has not adequately represented the economy. Very clearly, therefore, the same applies to corporate earnings as well.

6. Some Remarks and Preliminary Stats

It is important to mention once again that our conclusions so far have been based on less-than-elaborate statistical scrutiny. This, up till now, has simply entailed either visual contrasts or computing standard deviations relative to some best-fit curve to prove that data do indeed converge when mapped according to our proposed methodology. An example of this has been included in Figures 4a and 4b.

We now find it necessary now to conduct some preliminary statistics on how the mapping functions, $\Psi(R)$, $\Phi(R)$ and $\Xi(R)$, as described in Sections 2a and 2b, relate to one another. Respectively, these correspond to the price, the GDP and aggregated I/B/E/S corporate earnings forecasts.

The testing strategy is basic and goes as follows. First, we assume that there is a linear relationship between each of the functions – i.e.

$$\Psi(R) = a_0 + a_1\Phi(R), \qquad (17a)$$

$$\Psi(R) = b_0 + b_1\Xi(R) \qquad (17b)$$

and

$$\Phi(R) = c_0 + c_1\Xi(R) \qquad (17c)$$

where the coefficients $a_{0,1}$, $b_{0,1}$ and $c_{0,1}$ are regression constants. Second, in accordance with our initial conjecture, if the hypothesised "golden rule"[8] were to hold to within any reason, then

$$\Psi(R) = \Phi(R), \qquad (18a)$$

$$\Psi(R) = \Xi(R) \qquad (18b)$$

and

$$\Phi(R) = \Xi(R) \qquad (18c)$$

all of which are obtained from

$$a_0, b_0 \text{ and } c_0 = 0 \qquad (19a)$$

and

$$a_1, b_1 \text{ and } c_1 = 1 \qquad (19b)$$

Effectively, therefore, this so-called golden rule attempts to tie in together the financial

[7] The differences in this case are so obvious that they could be compared visually, but, never the less, qualitatively.

[8] The golden rule here will signify the situation where the three functions, $\Psi(R)$, $\Phi(R)$ and $\Xi(R)$, are, at any point in time, exactly equal to one another.

markets with the economy on a strict, one-to-one basis [i.e. $\Psi(R) = \Phi(R) = \Xi(R)$]. The outcome of this test, which involves running the regressions in 17a-c and assessing the significance of the null hypotheses in 19a-b, is presented in Table 1.

Before we proceed with any discussions, it is necessary to note that the US has been excluded from here. The reason again refers to Footnote 7 – very simply, a visual examination of the overlays in Figures 16-18 is sufficient to strongly reject either or both nulls, 19a and/or b. It is very clear in all these figures the absence of any strong one-to-one correspondence between $\Psi(R)$, $\Phi(R)$ and $\Xi(R)$. This is perhaps because the three appear to react differently to shocks.

In contrast to the US, the UK leads to an outcome that is apparently unlike. Here, for example, a quick visual assessment of the overlays portrayed in Figures 13-15 does indicate some amount of balance between the three elements.

The test statistics, notwithstanding, which are outlined in Table 1, provide a more detailed story. For instance, while the p-values indicate that the price index does not correspond that strongly with either, the economy [i.e. UK GDP] or corporate earnings, there does seem to be a strong 1:1 relationship between corporate earnings and the economy. A plausible explanation for this is that, on aggregate, the FTSE earnings forecasts do indeed represent a fair sample of the UK's economy. As for why the others don't match so well, it might be that (1) there is insufficient data, as I/B/E/S earnings forecasts for the FTSE 100 index became available only after 1987, and/or (2) our hypothesis of the golden rule may simply not be valid within the time frame tested.

To further help clarify the matter, we have included Figures 19a-b as well. Figure 19a is just a plot of the FTSE 100 aggregated-earnings forecast against the corresponding price index. Note the absence of any firm and/or conclusive relationship. Figure 19b, on the other had, portrays 19a as Ξ *versus* Ψ, which is just a re-plot of Figure 14. The solid, diagonal line displays our theorised conjecture in Equation 18c, whereas the dashed line is a linear best fit through the data. Simply stated, therefore, the corresponding p-value of 11%-15% in Table 1 is the level of confidence with which one could claim that this diagonal line is instead the best fit through the data in Figure 19b. As noted, the confidence level is not that high for this particular case.

Lastly, we find it necessary to mention a couple of difficulties that we encountered in testing our overall results. First, as detection and identification of structural tests comprise an integral part of this paper, we are not aware of any tests that check for the structural breaks within the context of our work. Recall that the traditional time-series-based methods for detecting structural breaks are not valid here purely because the mapping we incorporated is not time series. Secondly, as of yet, we have no theory to describe the shape of any of the functions, $\Psi(R)$, $\Phi(R)$ and $\Xi(R)$. The cubic-polynomial, best-fit curve shown in Figure 4b was used here for convenience to help us compute the standard deviation. With the above in mind, therefore, it goes without saying that (1) a more elaborate theory is needed to describe the factors that underlie the shapes of the above functions, and (2) a more robust and solid stronghold could be gained only after the results of our approach are fully subjected to more rigorous statistical scrutiny and testing. These, by themselves, provide scope and direction for further research along these lines.

7. Summary and Conclusions

An objective, and [hopefully] practical, approach to relative valuation has been proposed. The method, which entails a simple mapping, enables one to objectively compare the nominal GDP, corporate earnings and equity index relative to one another.

On applying this to the UK and US markets and economies, we reach certain conclusions, some of which are listed below.

(a) On the basis of this work, a comparison of the UK equity index, aggregated corporate earnings and the nominal GDP, as shown in Figures 13-15, suggests that the three have been, more or less, in balance with one another over the past 20 years. In other words, with the exception of a few quarters prior to the October, 1987, stock market crash, the UK shows no evidence of gross over/under pricing of any of these in relation to one another.

(b) A basic statistical test, focusing on the relationships between the price index, the aggregated earnings forecast and the economy, has led to some inconclusive results regarding the golden rule. While it was shown that the FTSE 100 corporate earnings compare well against the economy, any strong evidence of a one-to-one relationship between the [mapped] FTSE 100 price index with either, the corporate earnings and the UK GDP, appears to be lacking.

(c) The situation in the US is markedly different. Notwithstanding the lack of statistical tests here and relying more on visual comparisons, our work indicates that from 1980 to 1994, the three elements were roughly, and in terms of order of magnitude only, in balance with one another, as Figures 16-18 indicate. Post 1994, however, according to Figure 18 an upward shift in corporate earnings pushed the valuations above those implied by the nominal GDP. Moreover, during the same period, the equity price index has surpassed even the corporate earnings in terms of relative valuation [see Figure 17]. This peculiar behaviour provides clear evidence of what has been described as the "new economy," which presumably began in the US soon after 1994. There are certainly severe inconsistencies in the valuations, all which have been happening throughout this last regime.

(d) It is ironic that this so-called "new economy" in the US seems to apply to the equity market only, which consists of both, prices and corporate earnings, but not to the whole economy, as reflected here by the GDP. The US GDP, when mapped accordingly [see Figures 11, 16 and 18], appears to be immune from the impacts of this structural shift. This, perhaps, is owed to the GDP being too large to be measurably influenced by the effects of this new economy.

We finally conclude here by stressing once again that our approach here has nothing to do with forecasting. It simply manifests a long-term measure of relative valuation between an index, its aggregated corporate earnings and the GDP.

9. References

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Relationship	Implication	Available data range & no. of observations	Significance level[9,10] [p-value]
$\Psi(R) = \Phi(R)$	1:1 relationship between equity price index and economy	80:1 – 98:3 (75)	26%, 37%
$\Psi(R) = \Xi(R)$	1:1 relationship between equity price index and corporate earnings	87:3 – 99:4 (50)	11%, 15%
$\Phi(R) = \Xi(R)$	1:1 relationship between corporate earnings and economy	87:3 – 98:3 (45)	88%, 99%

Table 1 – Significance levels, in terms of p-values, for one-to-one relationships between the FTSE 100 price index, economy and the aggregated corporate earnings. These are represented by the symbols $\Psi(R)$, $\Phi(R)$ and $\Xi(R)$, respectively. The stats for the US have been excluded owing to reasons stated in Section 6.

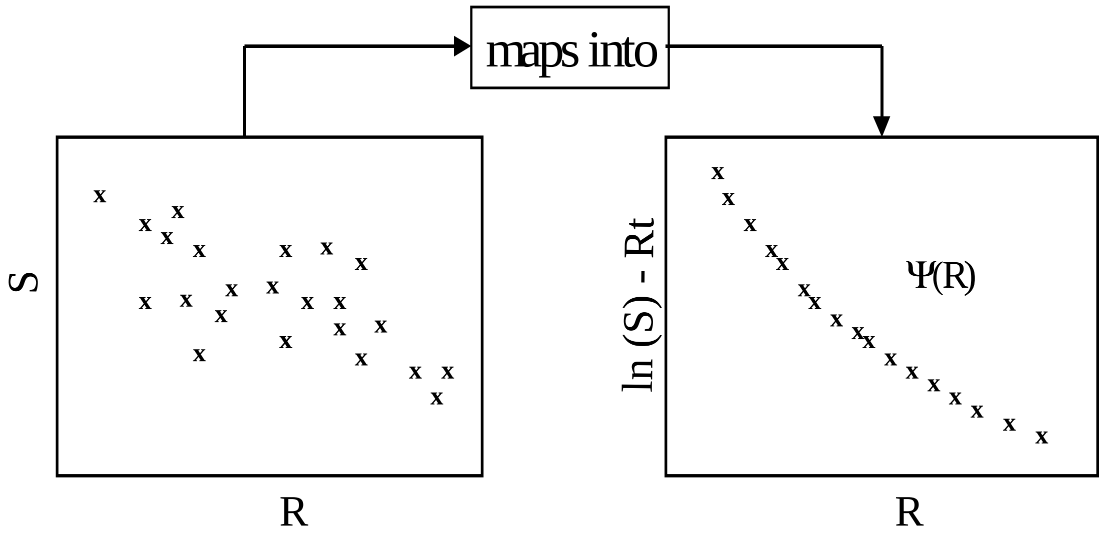

Figure 1 – Schematic diagram of convergence of data points under the proposed mapping.

[9] There are two p-values. The first refers to the null in Equation 19a and the second to 19b.
[10] The p-values are based on two-tailed tests around the Student-t distribution. The degrees of freedom are the number of observations [in column 3] minus 2.

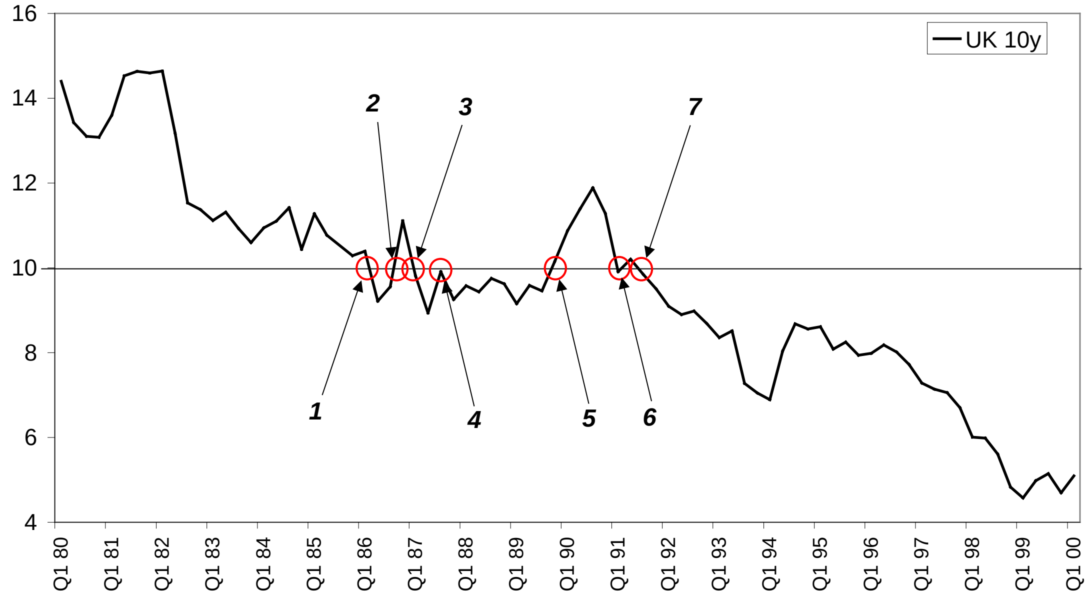

Figure 2 – Behaviour of the 10-year UK benchmark government bond yield between Q1-80 to Q1-00. The circled regions, which are numbered, are the locations where the yield crossed the 10% line during the past 20 years.

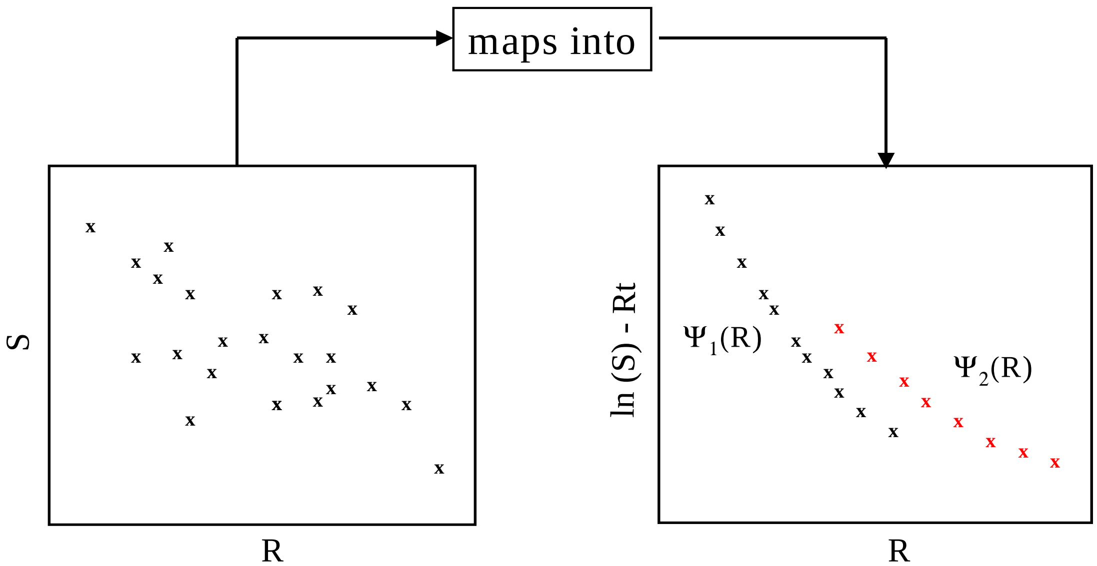

Figure 3 – Schematic diagram of how a regime shift manifests itself under the suggested mapping. A mapping of S *versus* R into $\ln S - Rt$ *versus* R leads to distinctive characteristic functions, depicted here by $\Psi_1(R)$ and $\Psi_2(R)$, each belonging to a separate regime.

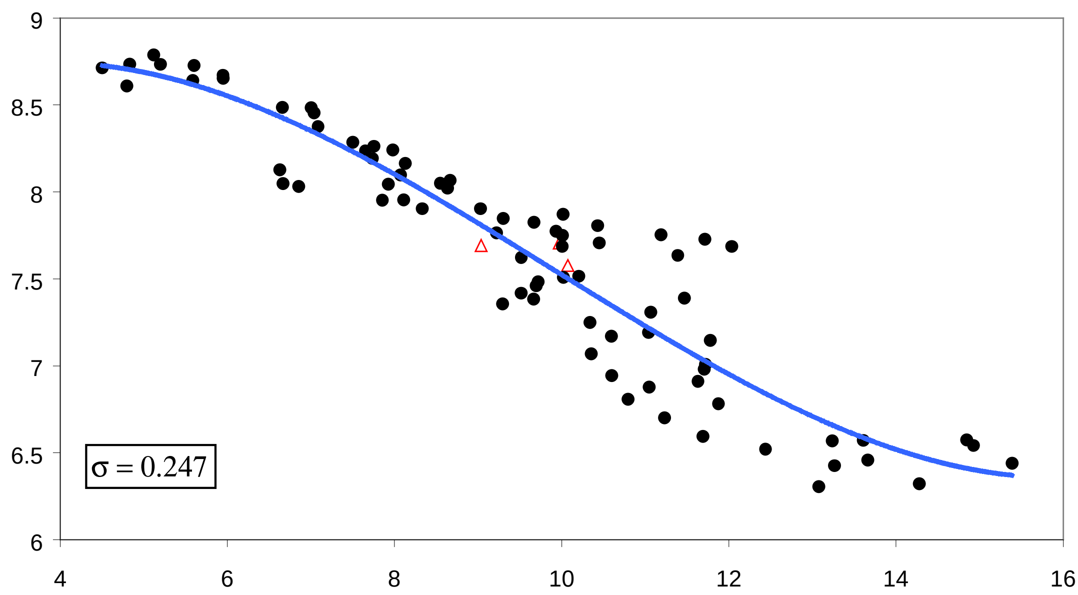

Figure 4a – The logarithm of the FTSE 100 price index, ln *S*, plotted against the nominal UK 10-year benchmark government bond yield. Note that this is not a time-series plot. Data range from Q1-80 to Q1-00. The triangles correspond to the 3 quarters immediately preceding the October 1987 crash. The line is a best-fit cubic polynomial and σ is the standard deviation of the data relative to it.

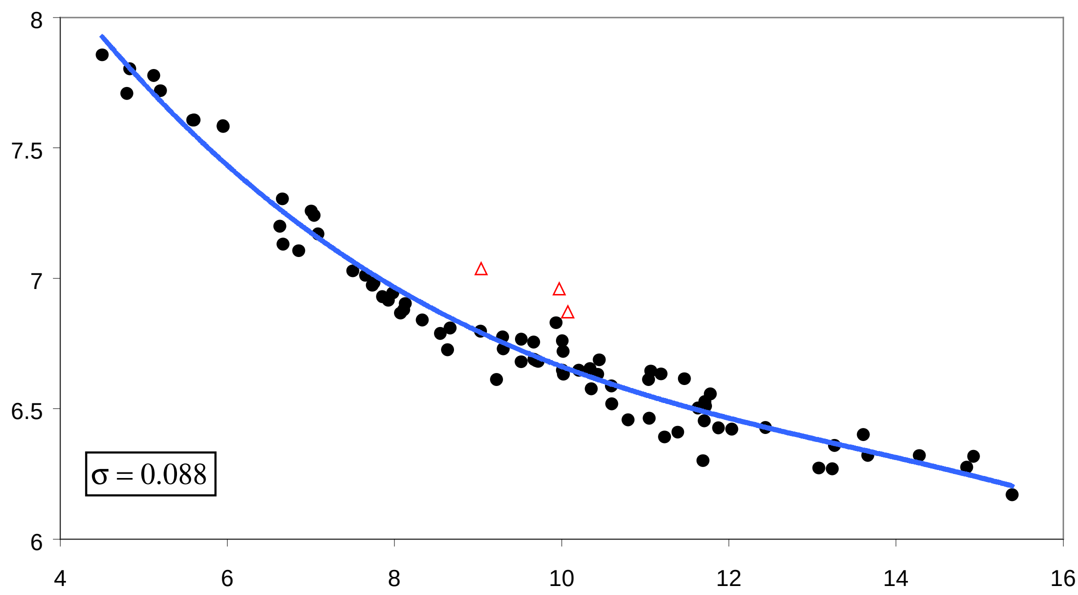

Figure 4b – The FTSE 100 price index in transformed co-ordinates, ln $S - Rt$, plotted against the nominal UK 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00. Note tightness of data relative to Figure 4a. The triangles correspond to the 3 quarters immediately preceding the October 1987 crash. The line is a best-fit cubic polynomial and σ is the standard deviation of the data relative to it.

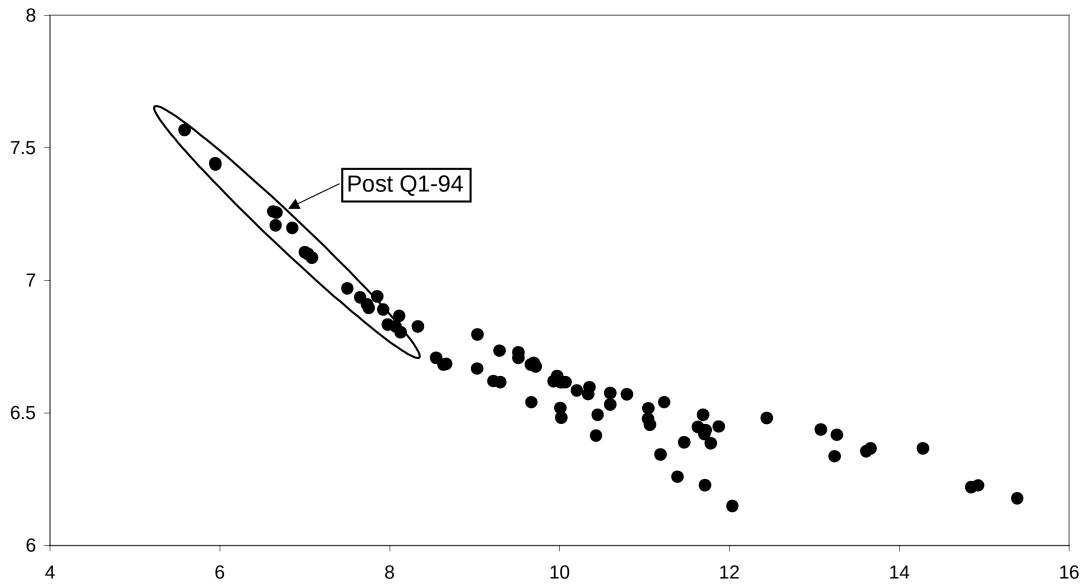

Figure 5 – The UK nominal GDP in transformed co-ordinates plotted against the UK 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00. The circled region presumably belongs to a distinctive structural regime.

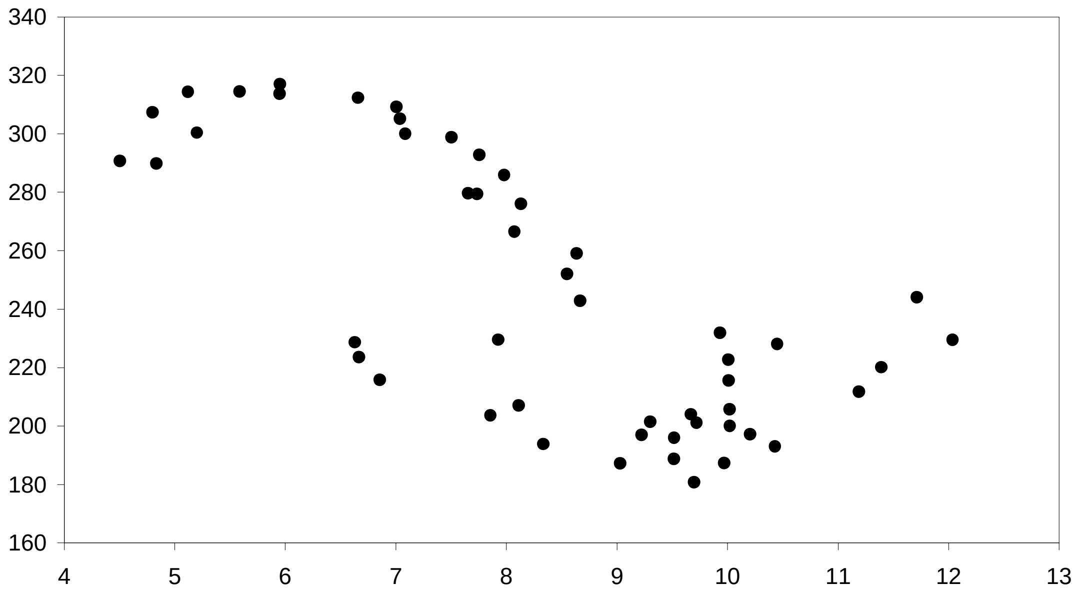

Figure 6a – The FTSE 100 corporate earnings, as is, plotted against the nominal UK 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00.

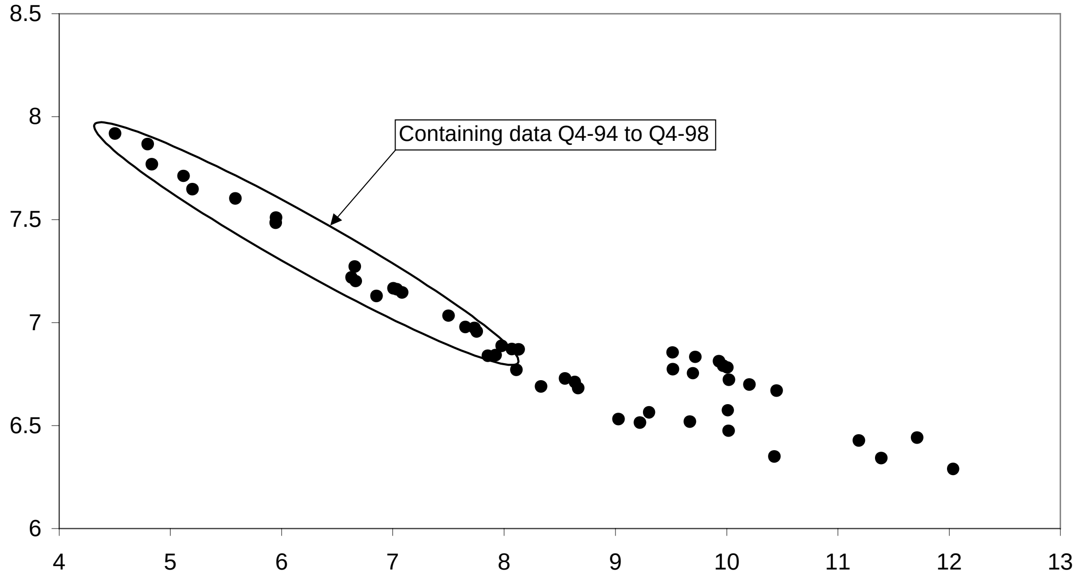

Figure 6b – The FTSE 100 corporate earnings in Figure 6a plotted in transformed co-ordinates against the nominal UK 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00. The circled region presumably belongs to a distinctive structural regime.

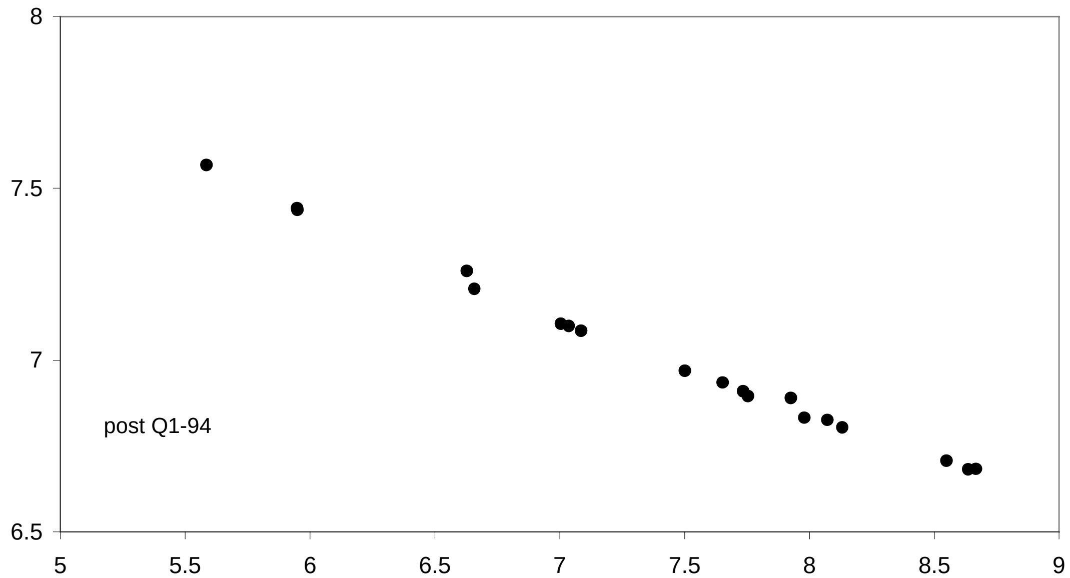

Figure 7 – Expanded view of the structural regime highlighted in Figure 5.

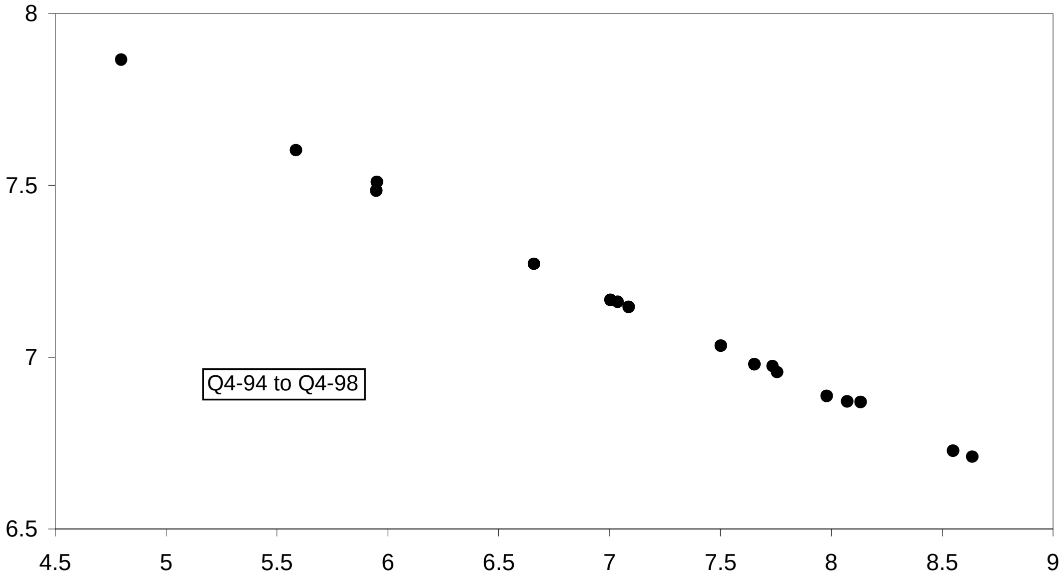

Figure 8 – Expanded view of the structural regime highlighted in Figure 6.

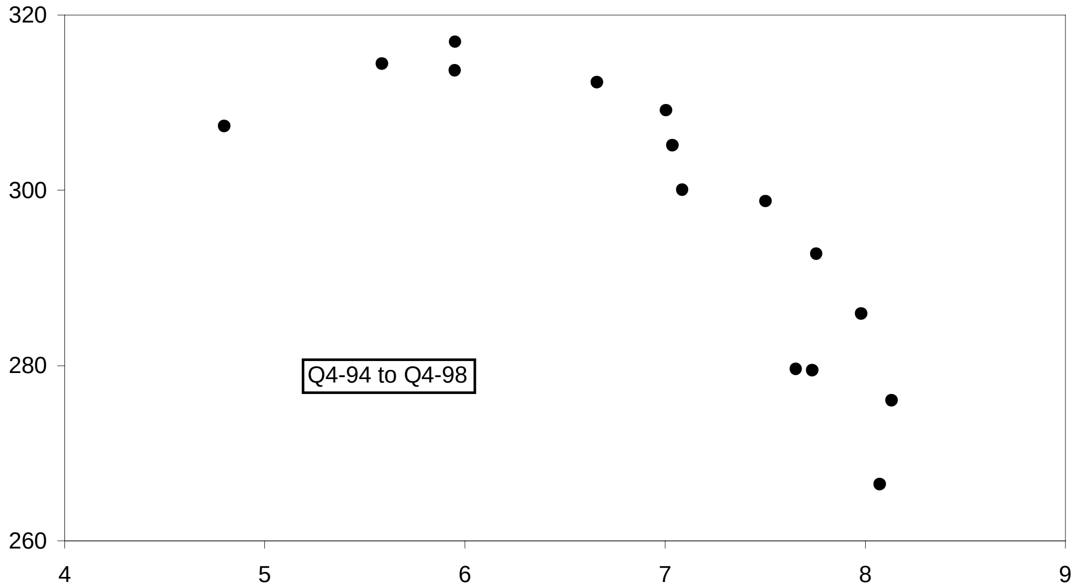

Figure 9 – The same structural regime shown in Figure 8, but plotted as is – i.e. e_f *versus* the nominal 10-year yield. This is a subset of Figure 6a. Note the loss of data convergence in comparison with its counterpart in Figure 8.

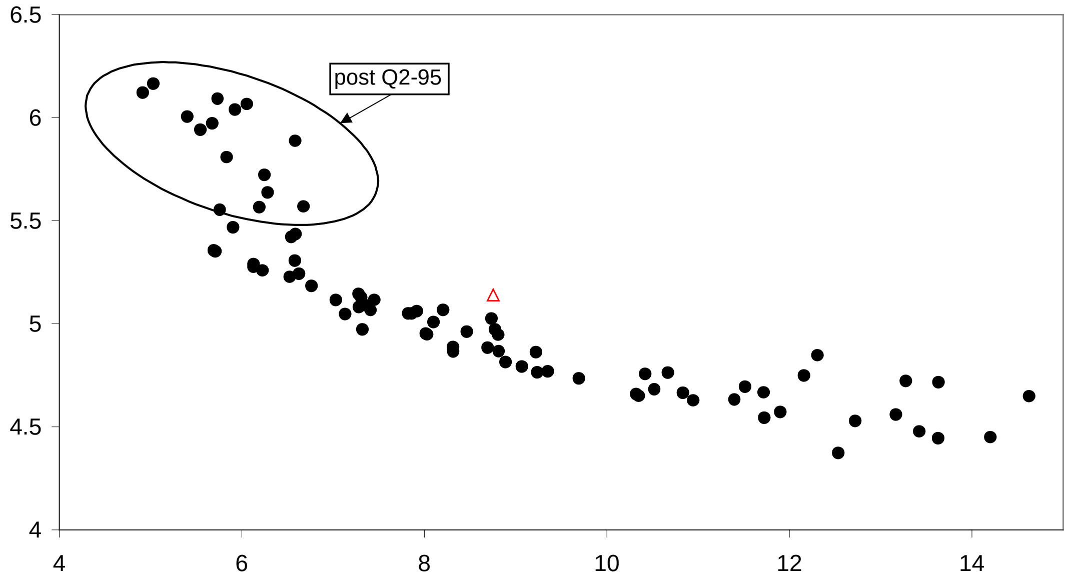

Figure 10 – The S&P 500 price index in transformed co-ordinates plotted against the nominal US 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00. The triangle belongs to the quarter immediately preceding the October 1987 crash, while the circled region highlights a distinct structural regime covering the more recent time frame.

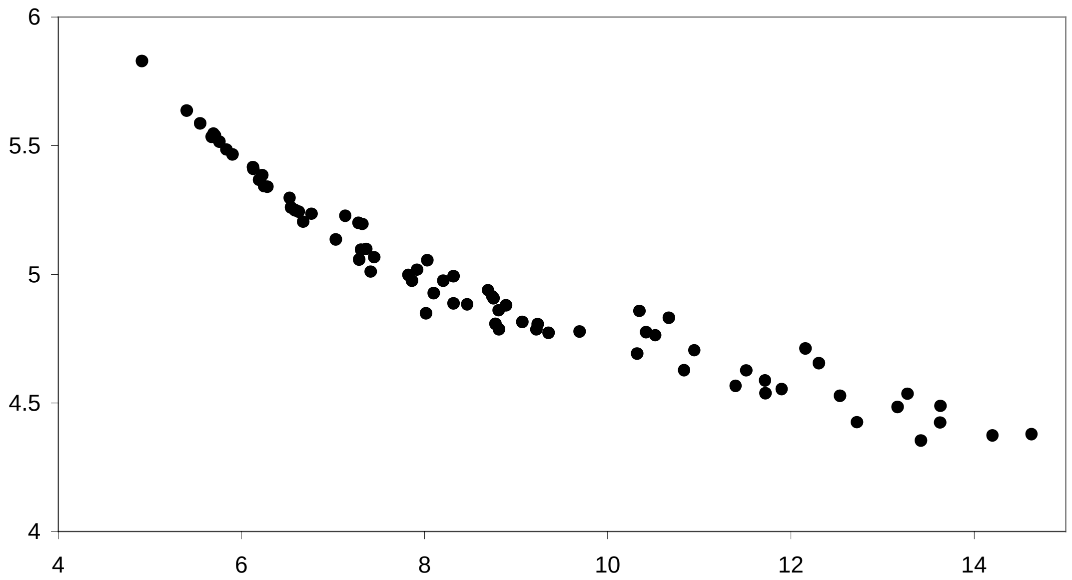

Figure 11 - The US nominal GDP in transformed co-ordinates plotted against the nominal US 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00.

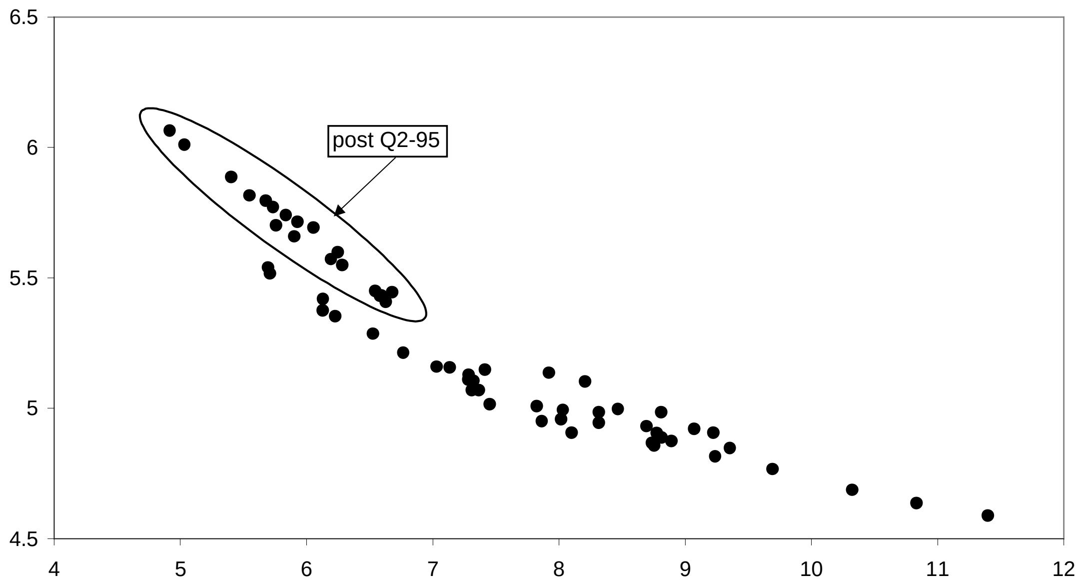

Figure 12 - The S&P 500 corporate earnings in transformed co-ordinates plotted against the nominal US 10-year benchmark government bond yield. Data range from Q1-80 to Q1-00. The circled region presumably belongs to a distinctive structural regime.

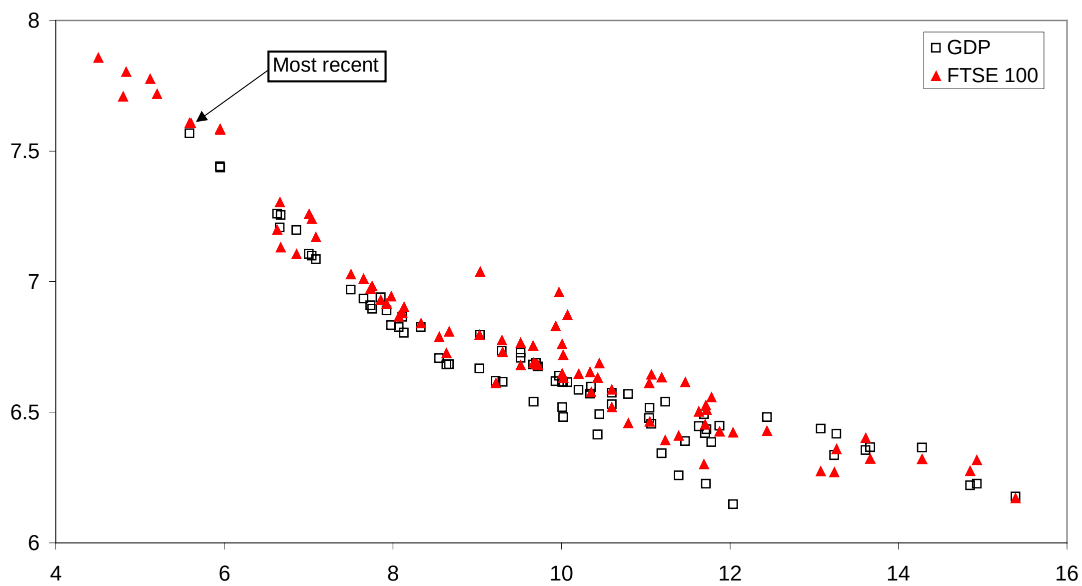

Figure 13 – Overlay of Figure 4 on 5, showing the FTSE 100 equity price index in comparison with the UK nominal GDP. Both parameters are in transformed co-ordinates. The most recent data point is highlighted.

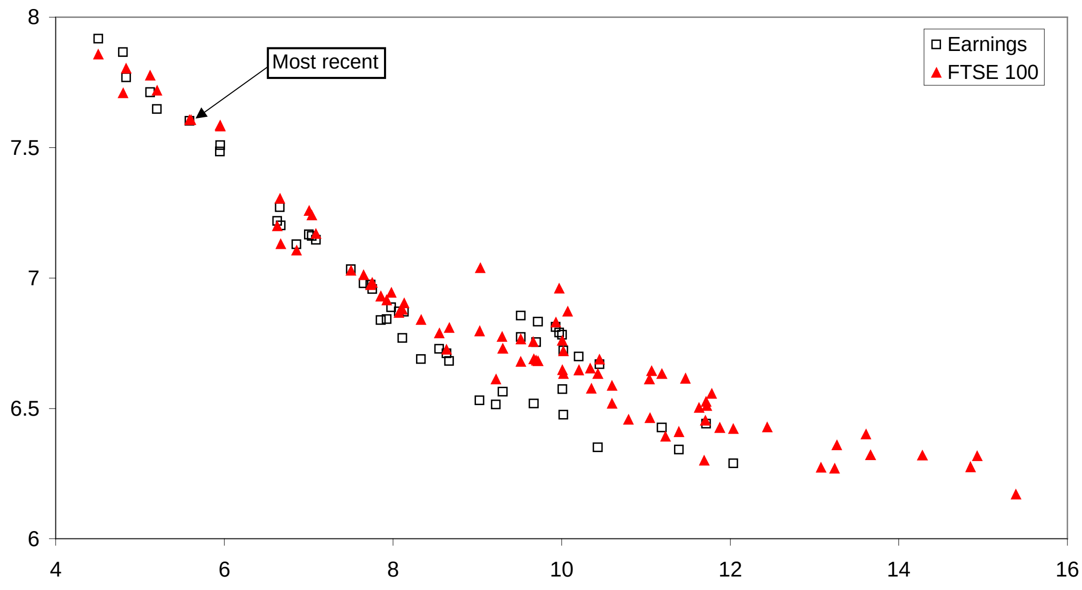

Figure 14 – Overlay of Figure 4 on 6, showing the FTSE 100 equity price index in comparison with the corporate earnings. Both parameters are in transformed co-ordinates. The most recent data points are highlighted.

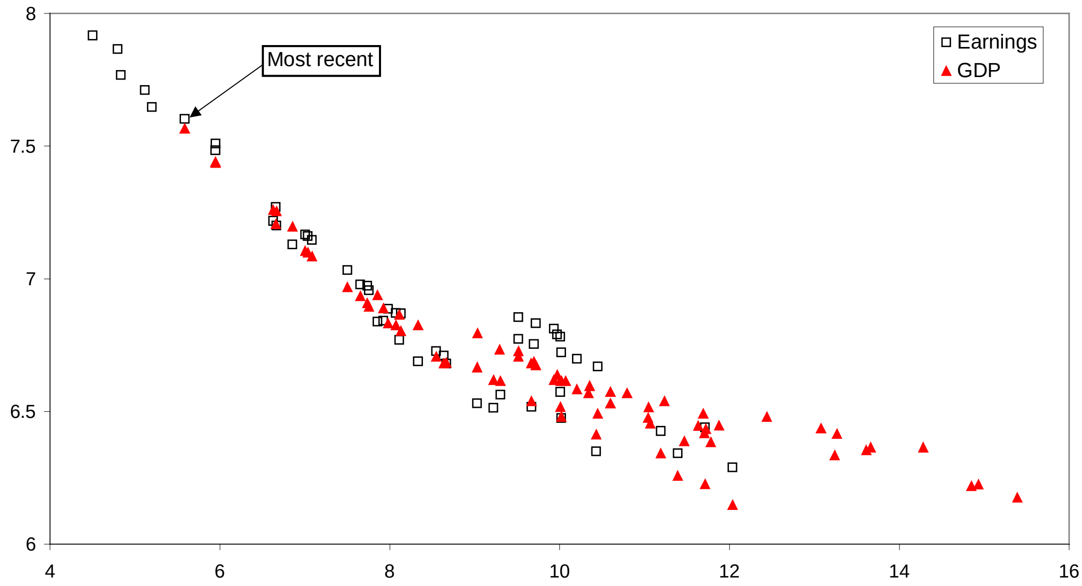

Figure 15 – Overlay of Figure 5 on 6, comparing the UK nominal GDP with the FTSE 100 earnings. Both parameters are transformed. The most recent data point is highlighted.

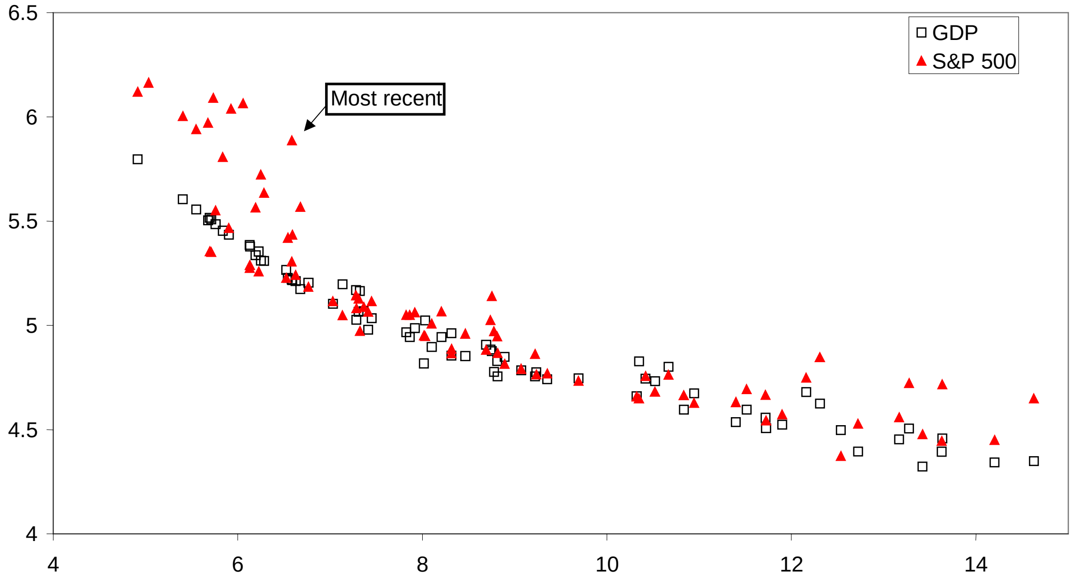

Figure 16 – Overlay of Figure 10 on 11, showing the S&P 500 equity price index in comparison with the US nominal GDP. Both parameters are in transformed co-ordinates. The most recent data point is highlighted.

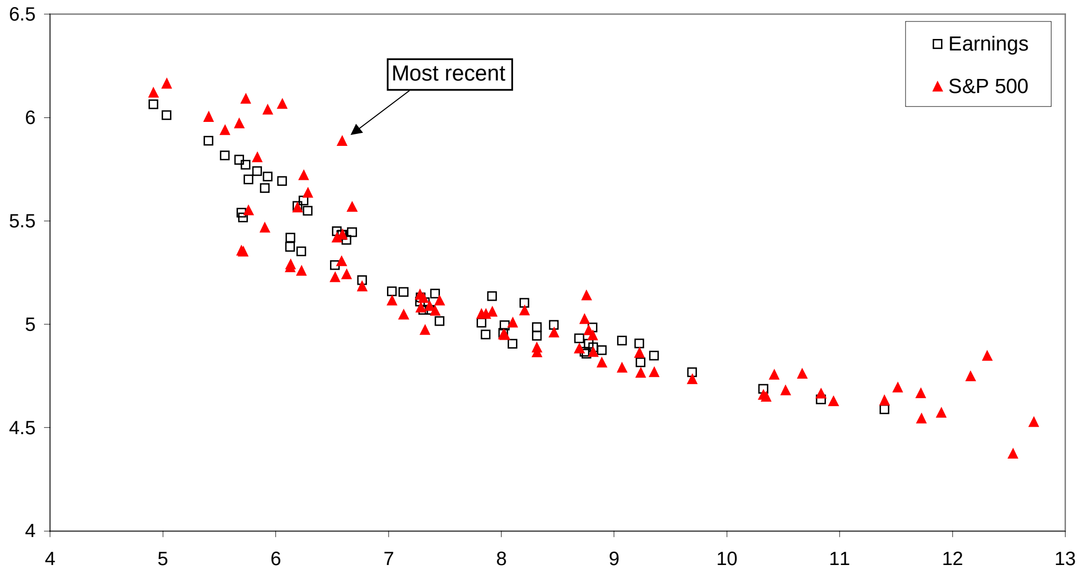

Figure 17 – Overlay of Figure 10 on 12, showing the S&P 500 equity price index in comparison with the corporate earnings. Both parameters are in transformed co-ordinates. The most recent data point is highlighted.

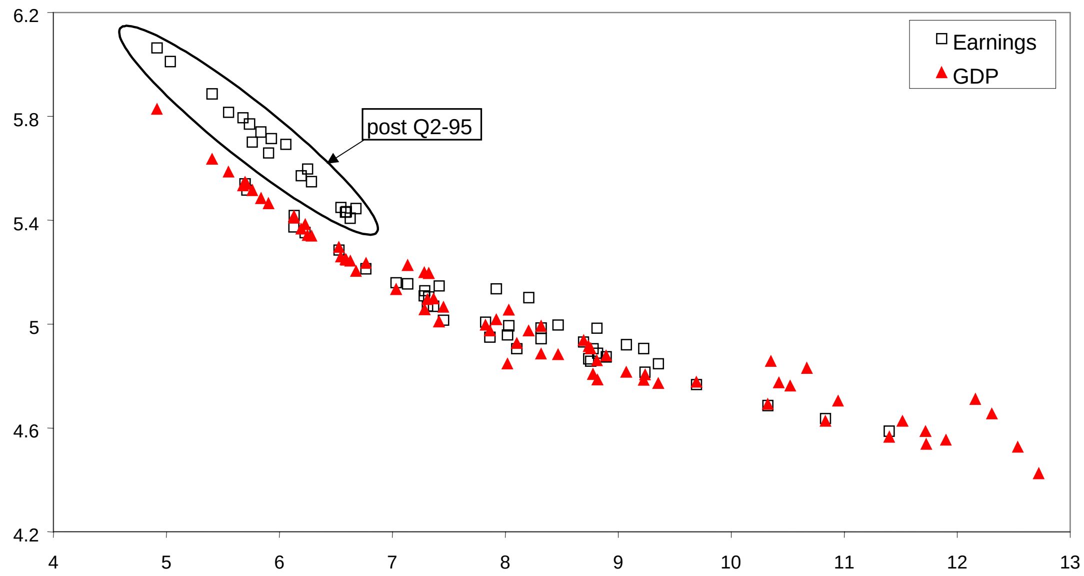

Figure 18 - Overlay of Figure 11 on 12, showing the US nominal GDP in comparison with the S&P 500 corporate earnings. Both parameters are transformed. The structural regime circled in Figure 12 is again highlighted here.

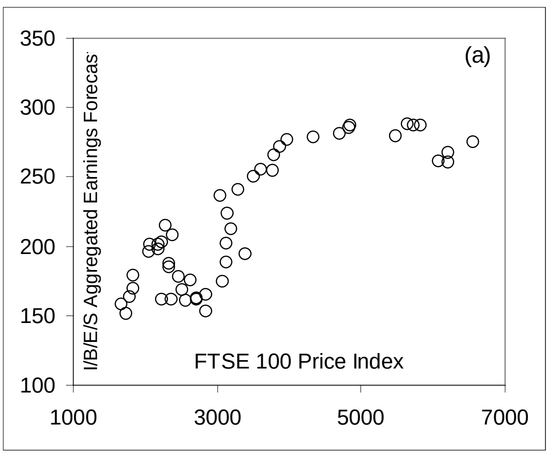

Figure 19a – The I/B/E/S aggregated earnings forecast for the FTSE 100 *versus* the corresponding price index. Note the absence of any firm and conclusive relationship.

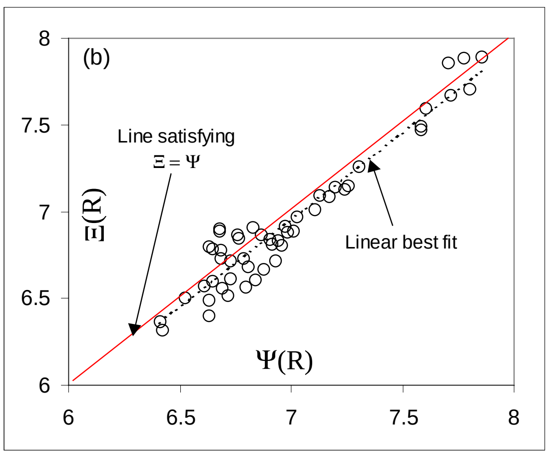

Figure 19b – Earnings, Ξ, *versus* the price index, Ψ, both in transformed co-ordinates. This is simply a re-plot of Figure 14. The dashed line is a linear best fit, whereas the solid diagonal satisfies one of our theoretical conjectures, namely $\Xi = \Psi$ [see Equation 18b].